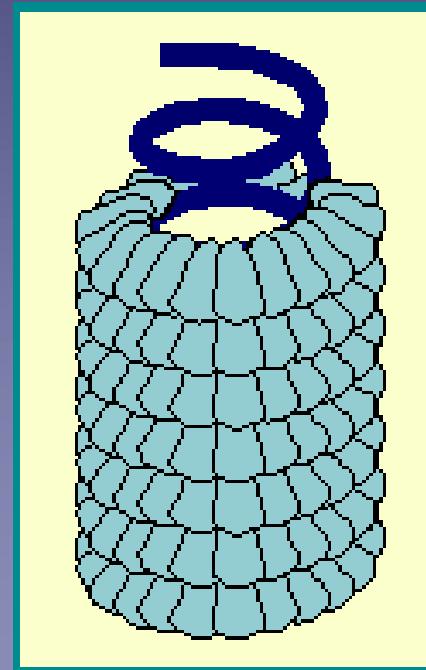
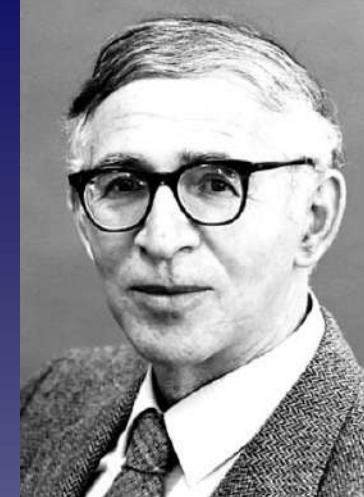
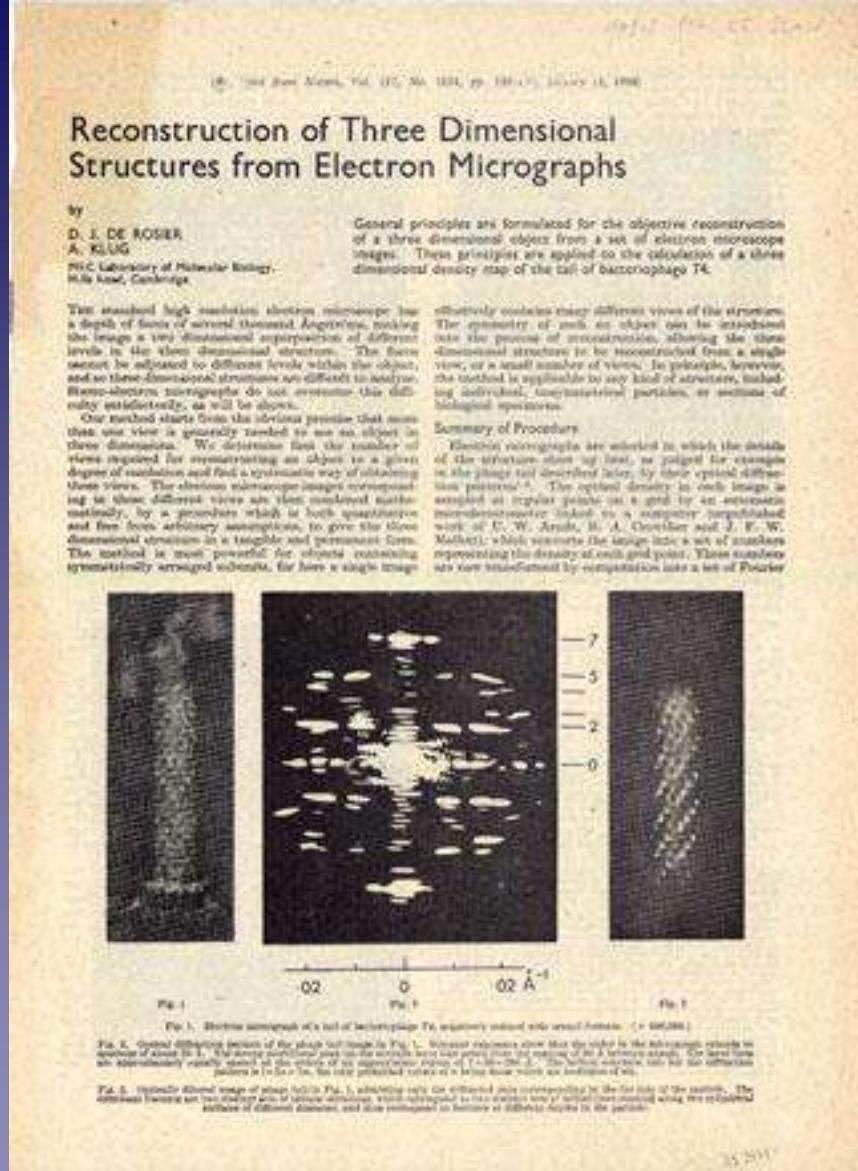


# **3D Reconstruction of Helical Specimens**



# Many Biological Specimens have helical symmetry

- DNA
- $\alpha$ -Helix
- Viruses (TMV)
- Actin filaments
- Myosin filaments
- Microtubules
- Bacterial Flagella
- Protein-lipid tubes



De Rosier & Klug. Nature 217: 130-134 1968

# Topics

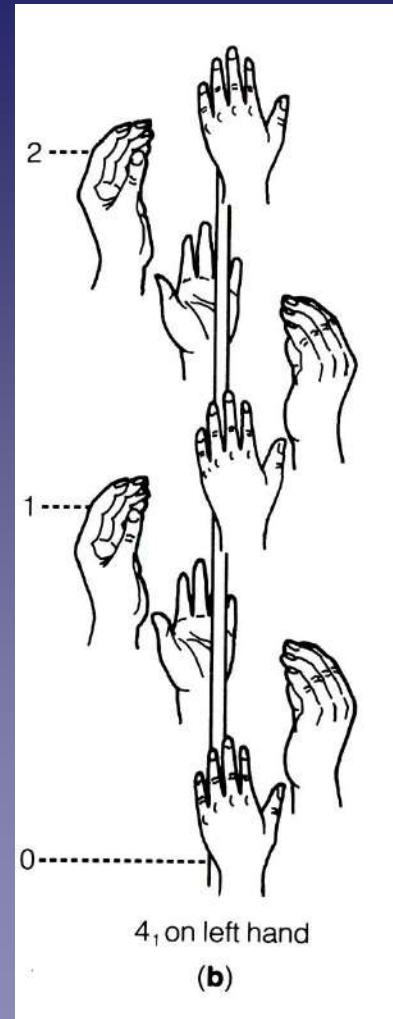
- History
- Helix definition
- Fourier Transform of a helix
- Fourier-Bessel helical 3D reconstruction
- Determining sample helical symmetry (twist & rise, selection rule)
- Real space/single particle helical 3D reconstruction
- Resolving asymmetric features in helical assemblies
- Examples.

# Helical Symmetry

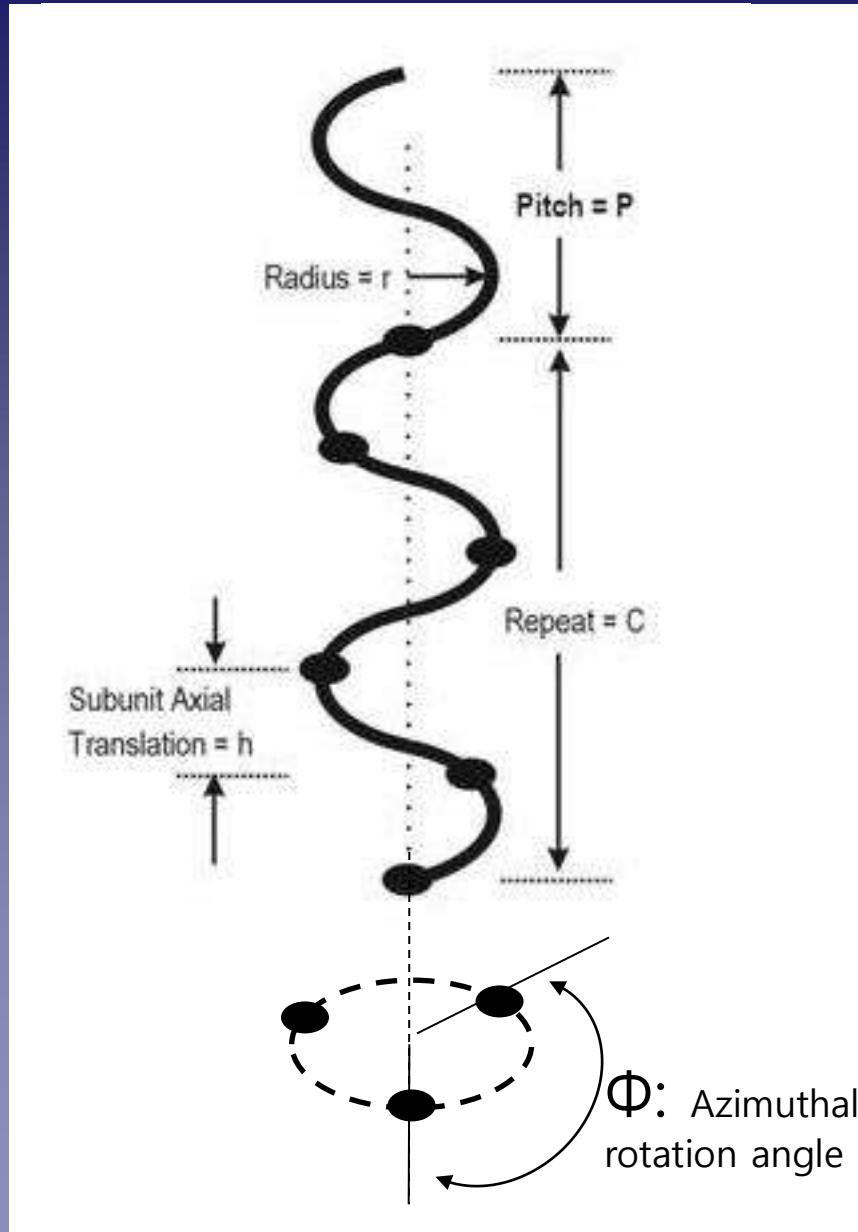
Combining the symmetry operation of **translation and rotation** (screw) produces a **helix**

## Possible Symmetry operations:

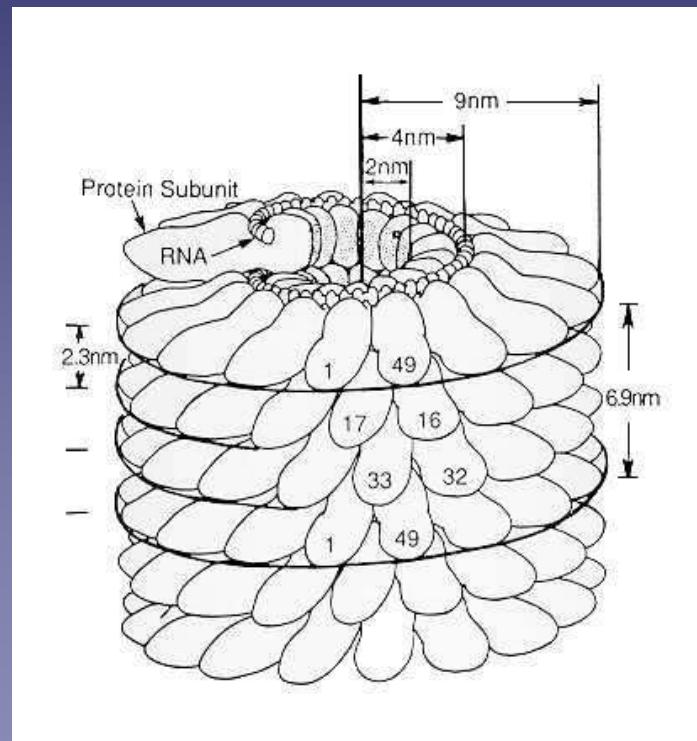
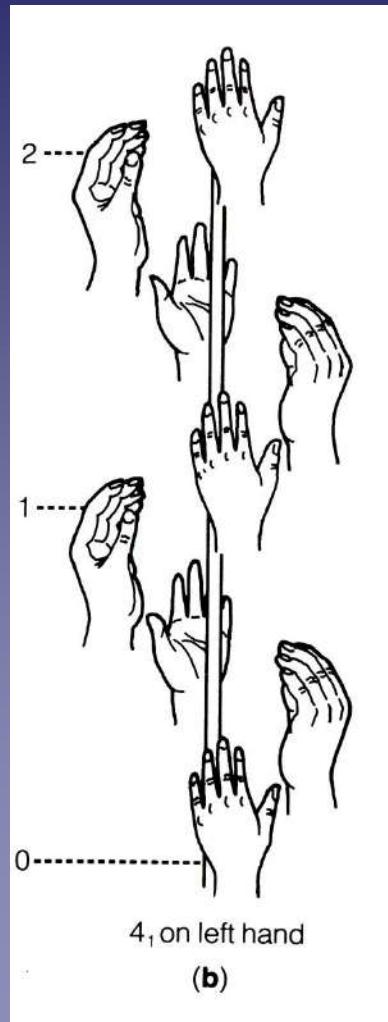
- Screw.
- n-fold rotation about axis.
- 2-fold rotation perpendicular to axis.



# Parameters of a Helix

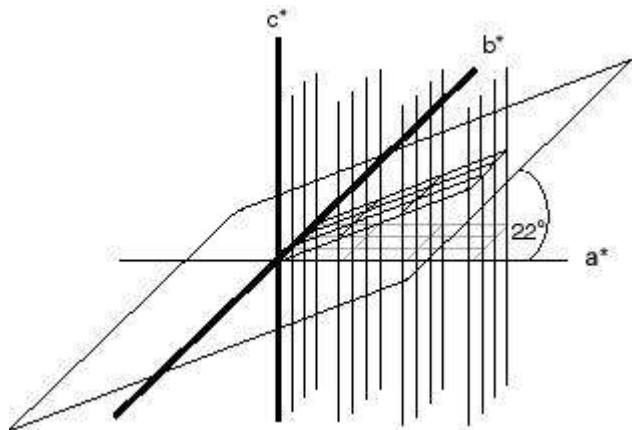


Helices give several orientation views of the asymmetric unit from a single view direction

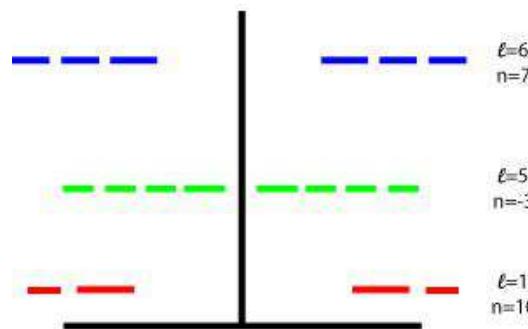


# No Missing Cone !

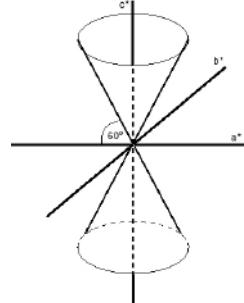
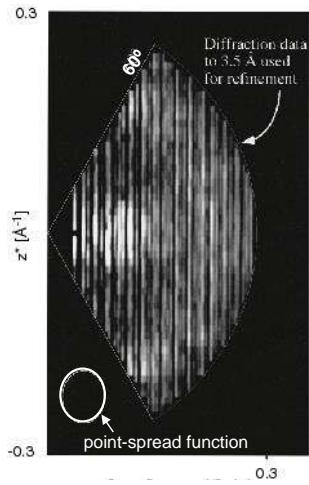
2D crystals



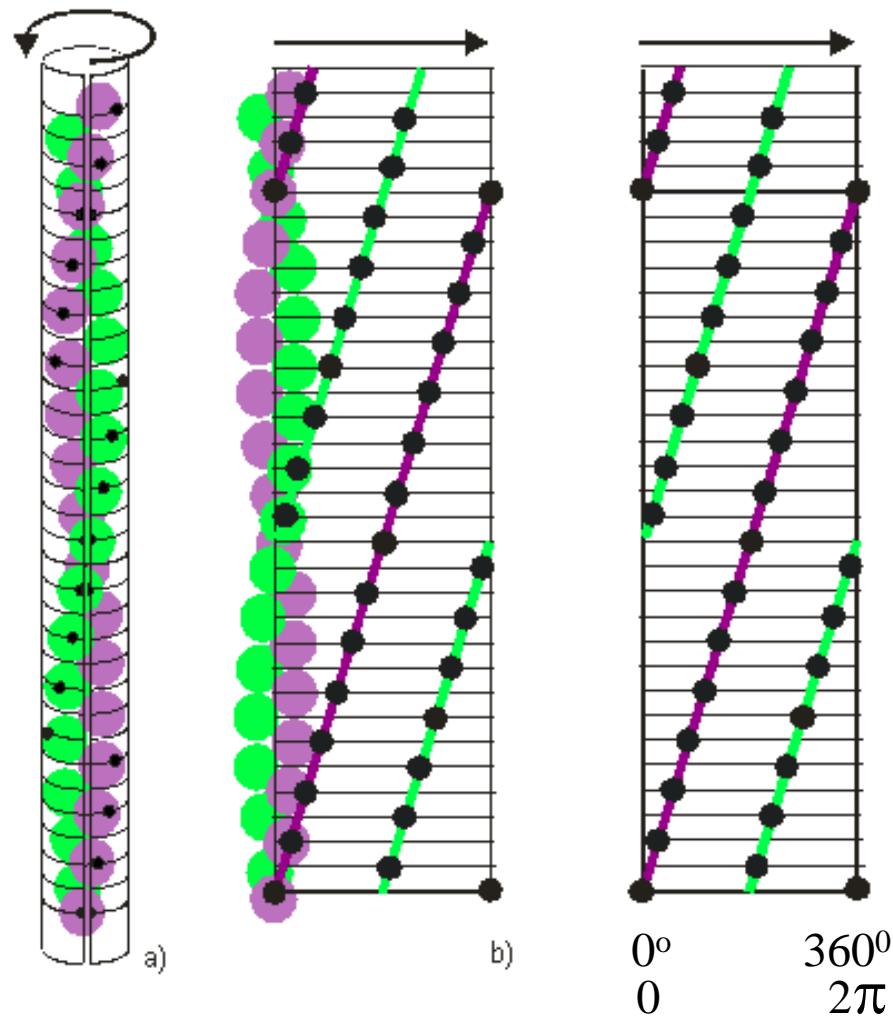
Helical crystals



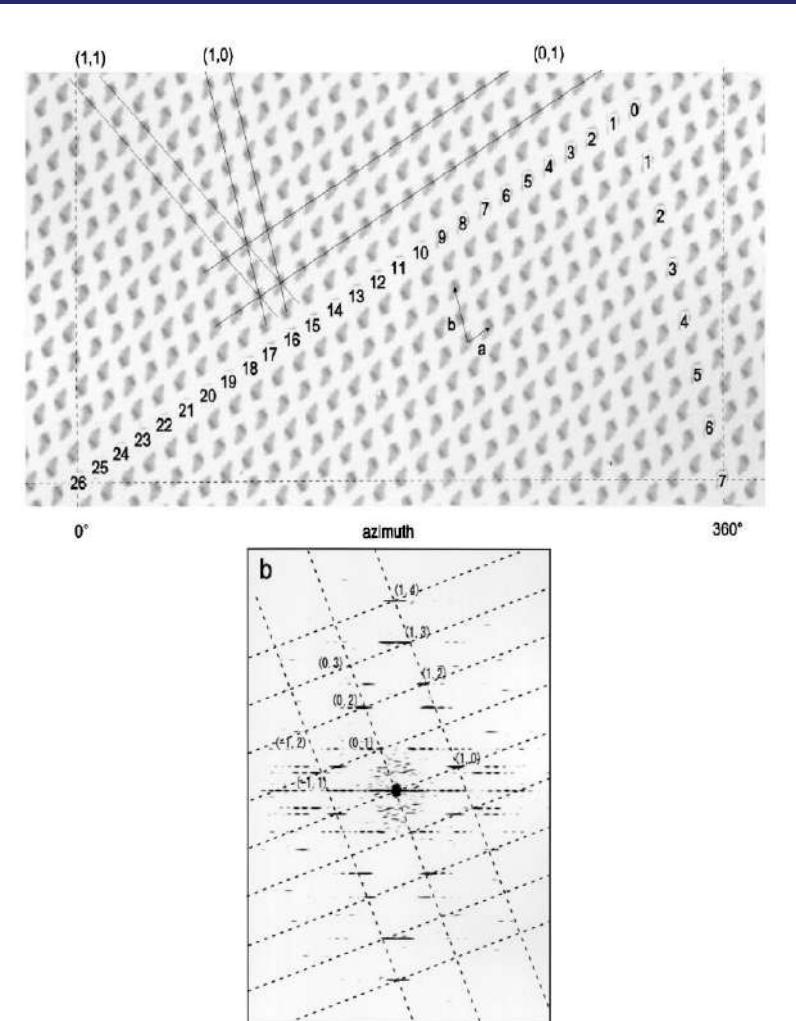
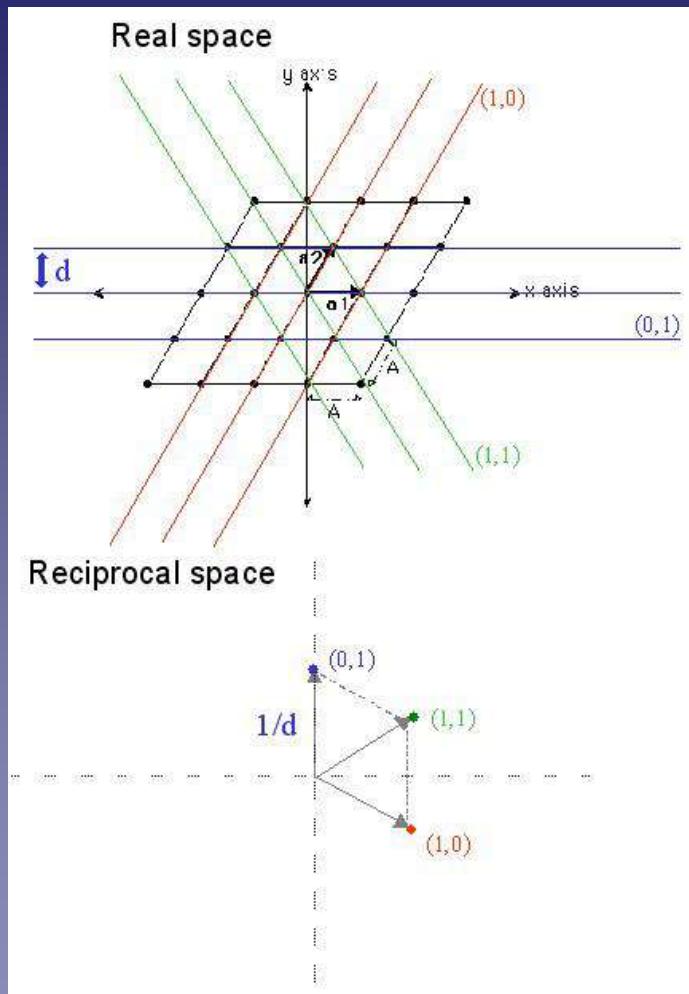
missing cone



# The Helical Lattice Radial Projection

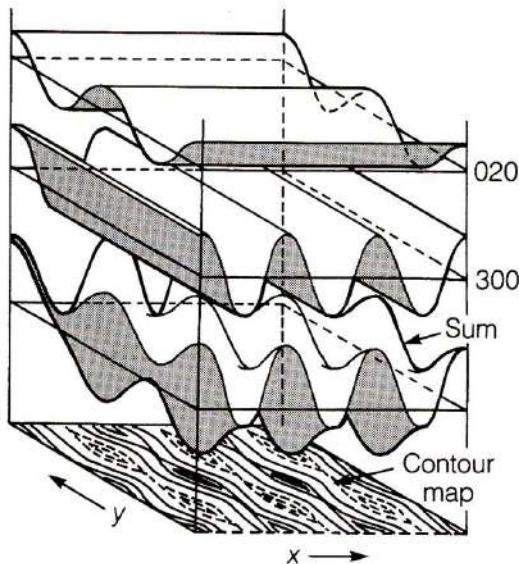
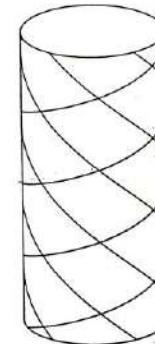
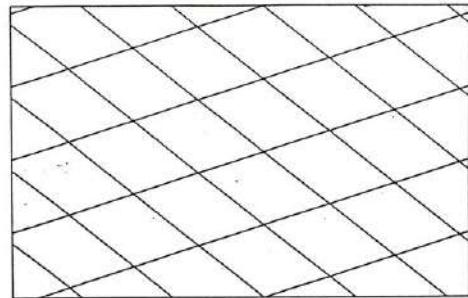


# Analogy between 2D lattices and Helical Lattices



From: Toyoshima (2000) Ultramicroscopy 84: 1-14

# Analogy between 2D Fourier synthesis and Fourier-Bessel helical synthesis



Summation of 2D waves to produce a 2D density map. (From Jeffery 1972)



A helical wave

# The Fourier Transform of a Helix

$$T(R, \varphi, n/P) = J_n(2\pi Rr) \exp [in(\varphi + \frac{1}{2}\pi)]$$

Cochran, Crick & Vand 1952

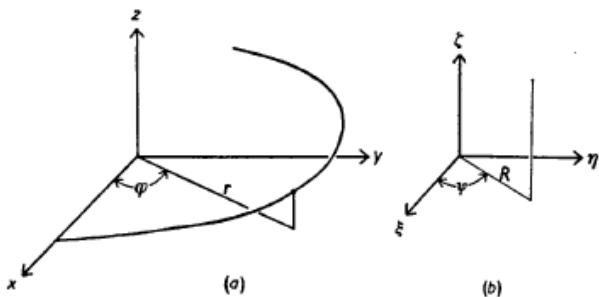


Fig. 1. (a) Cartesian ( $x, y, z$ ) and cylindrical-polar ( $r, \varphi, z$ ) coordinates of a point on a helix. (b) Corresponding coordinates of a point in reciprocal space.

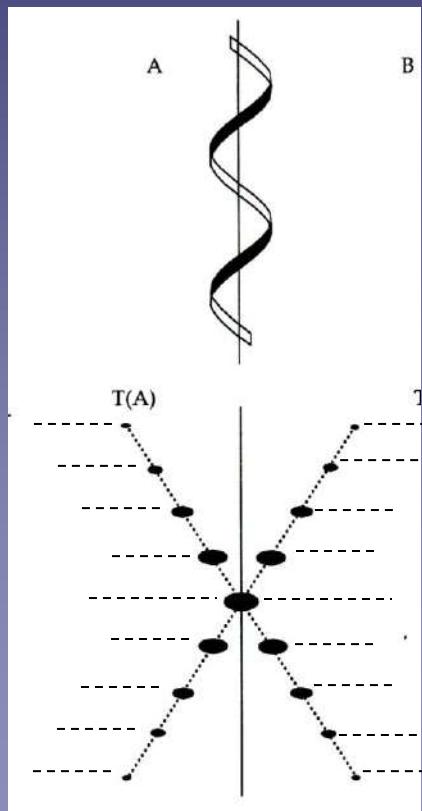
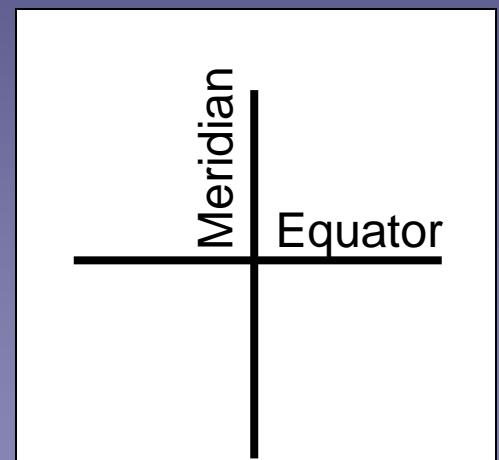
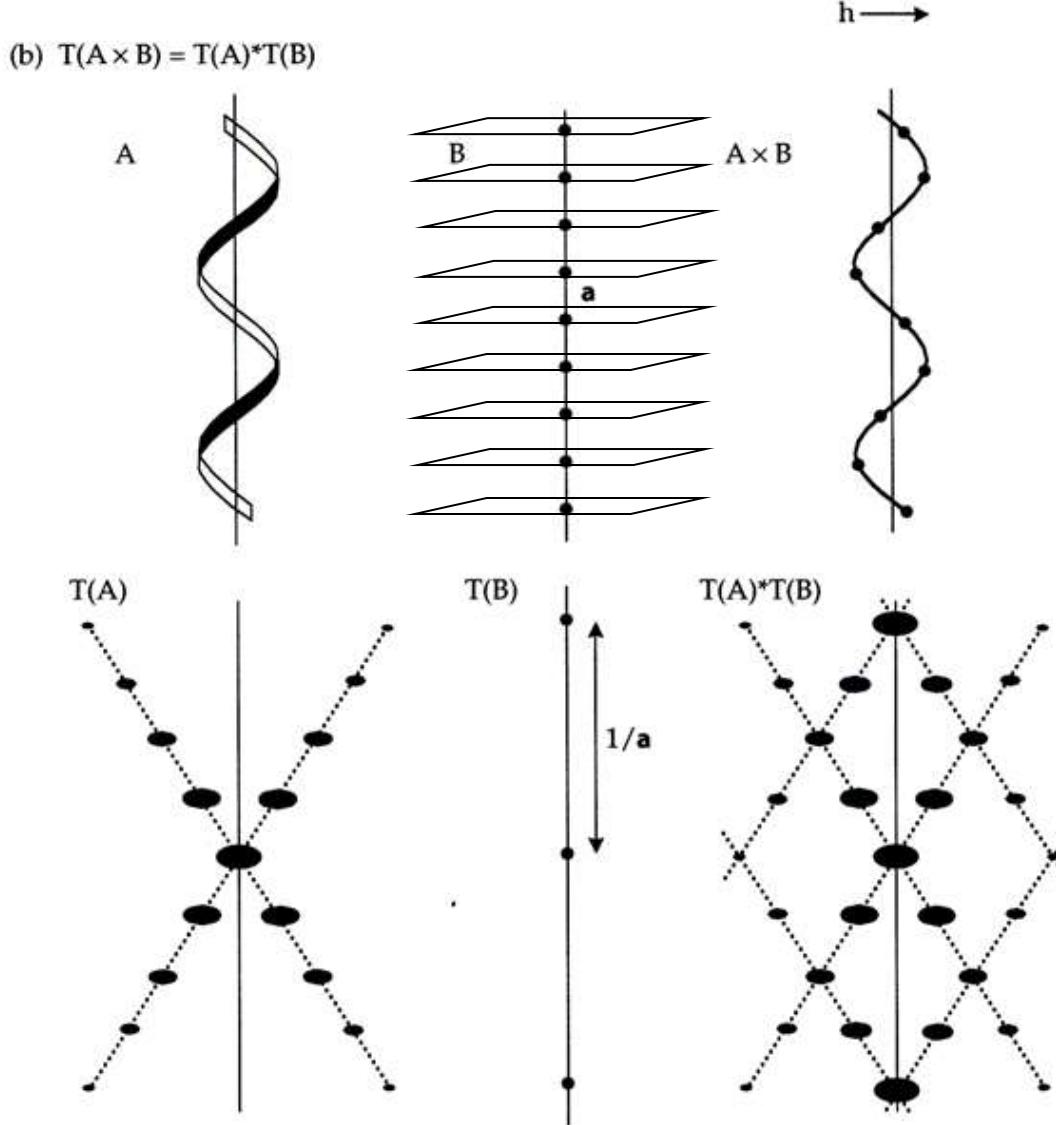


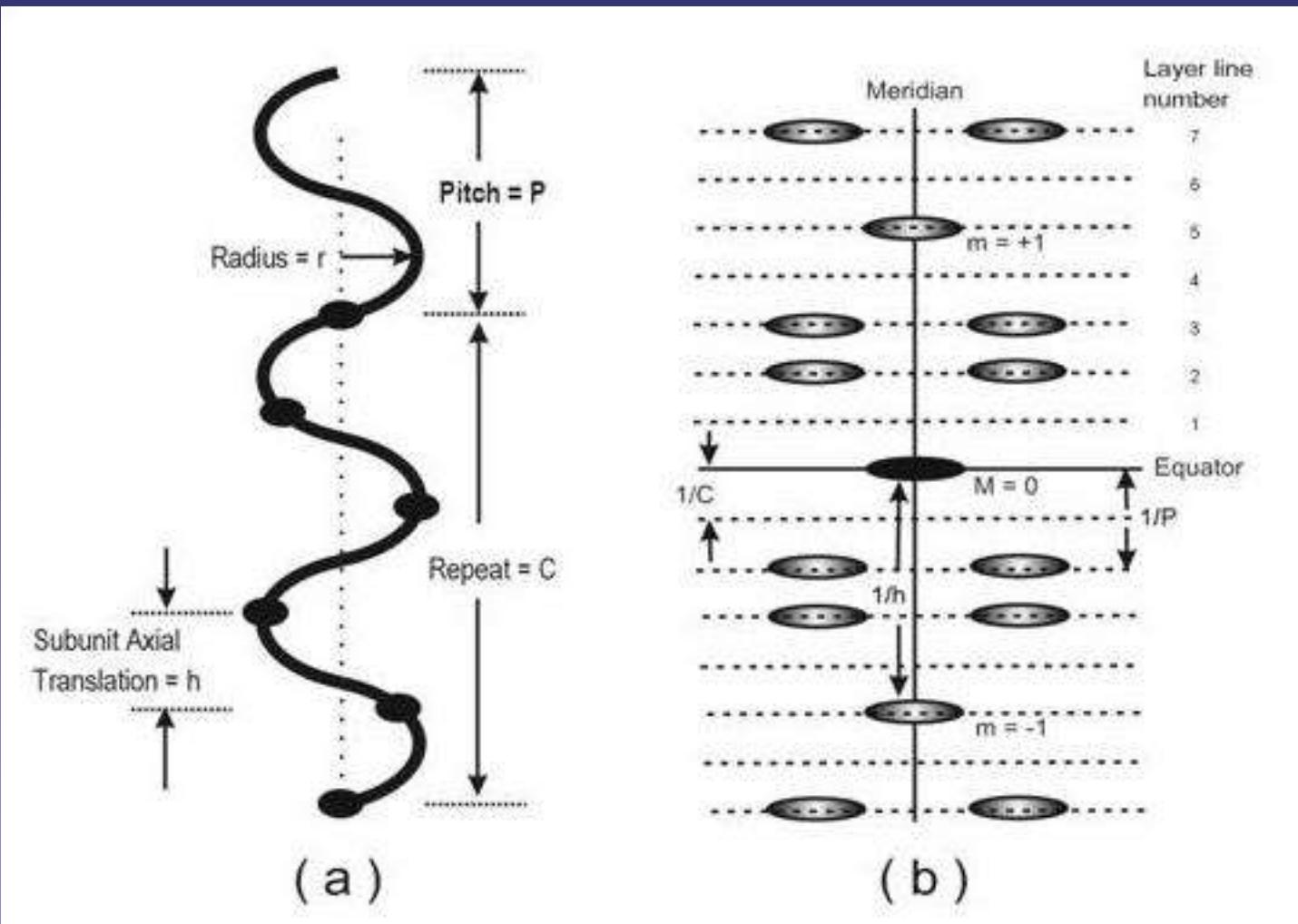
Fig. 2. Illustration of Bessel functions. (Reproduced by kind permission of the publishers from *Tables of Functions* by Jahnke & Emde. New York: Dover Publications.)



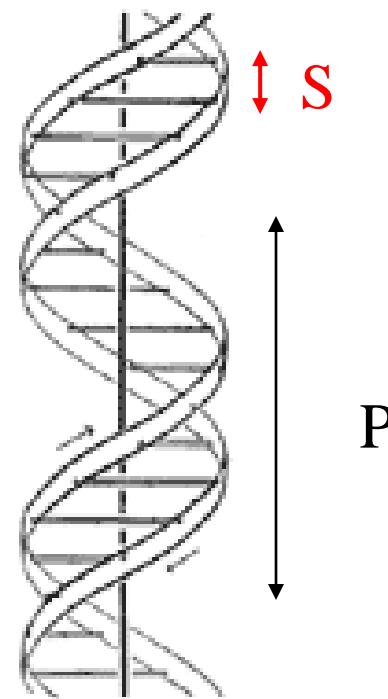
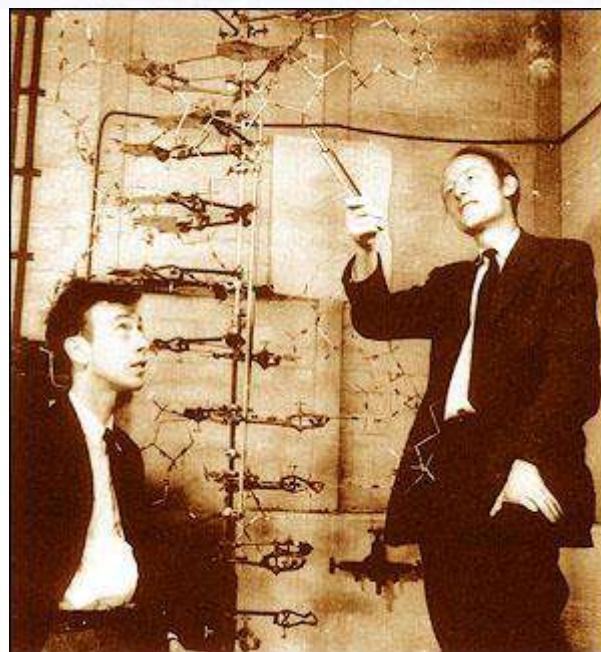
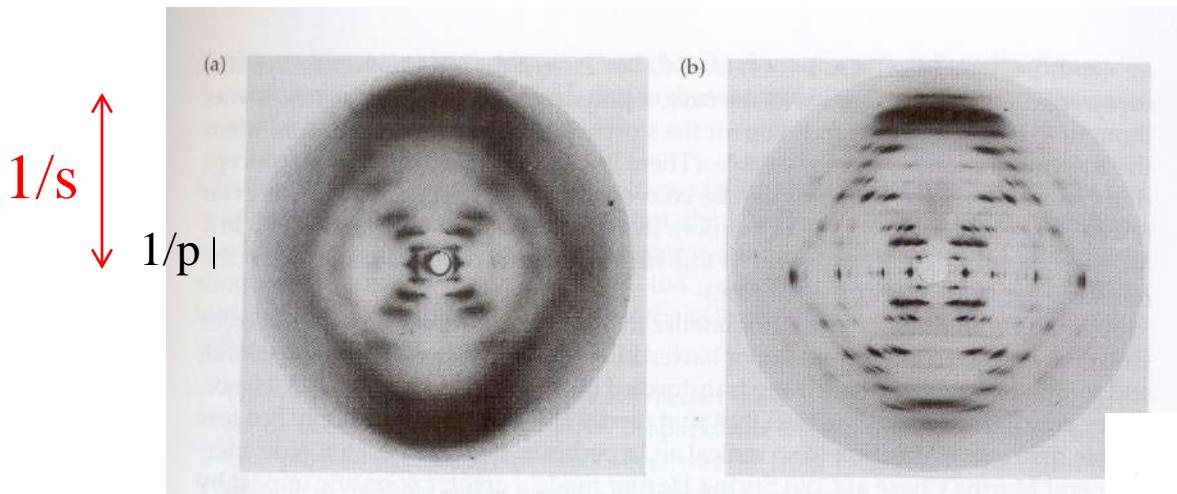
# The Transform of a discontinuous helix



# A helix and its corresponding Fourier Transform (Power Spectrum)



# The DNA Structure



# Helical 3D reconstruction Using the Fourier-Bessel Method

DeRosier & Moore J. Mol. Biol. 52:335 1970

Fourier Transform

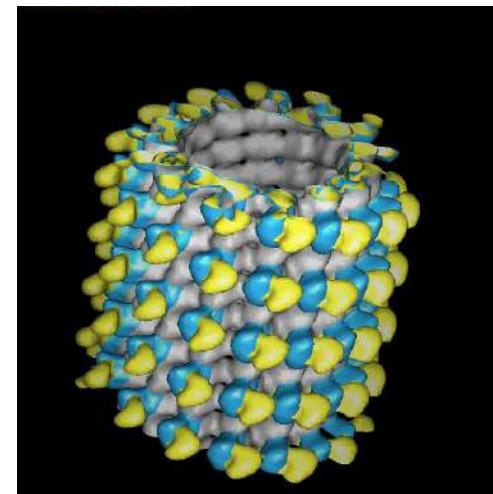
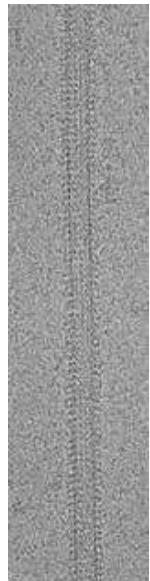
Reciprocal Space Function

$$F(R, \Phi, l/c) = \sum_n G_{n,l}(R) \exp [in(\Phi + \pi/2)]. \quad (1)$$

$$\text{Selection rule} \quad l = tn + um \quad (2)$$

Real space function (structure)  $\rho(r, \phi, z) = \sum_l \sum_n g_{n,l}(r) \exp(in\phi) \exp(-2\pi ilz/c) \quad (3)$

$$g_{n,l}(r) = \int G_{n,l}(R) J_n(2\pi Rr) 2\pi R dR$$



# Helical 3D reconstruction Using the Fourier-Bessel Method

DeRosier & Moore J. Mol. Biol. 52:335 1970

358

D. J. DEROSIER AND P. B. MOORE

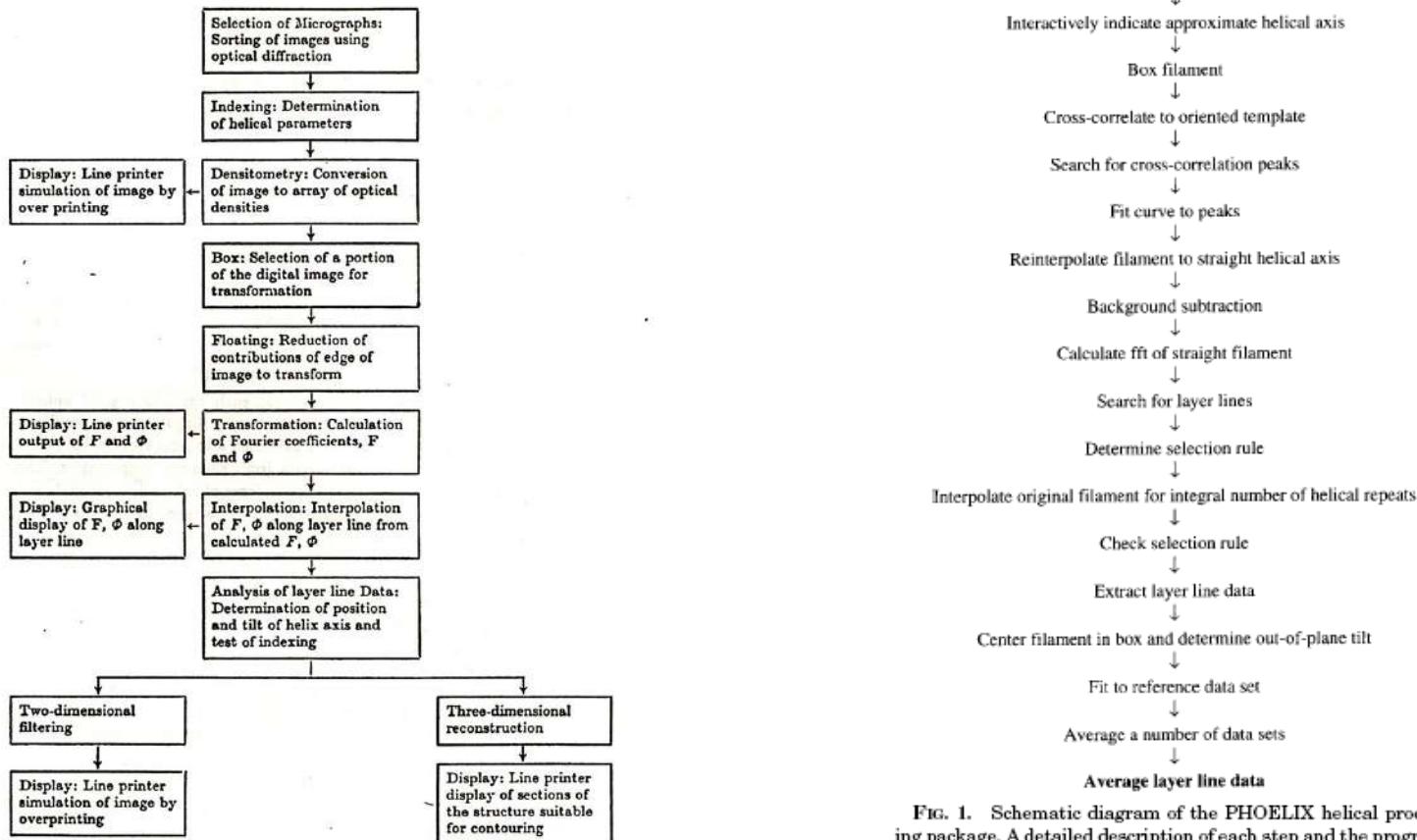
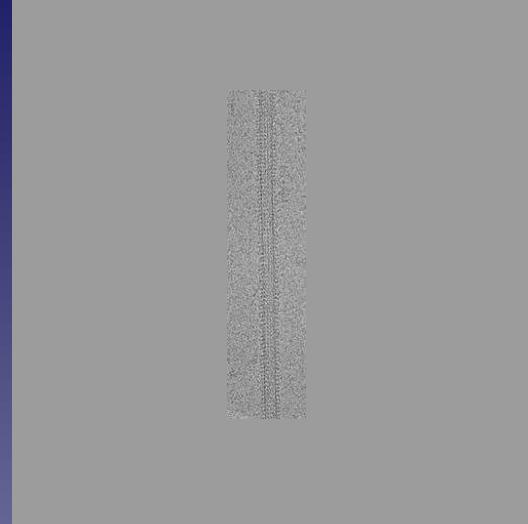
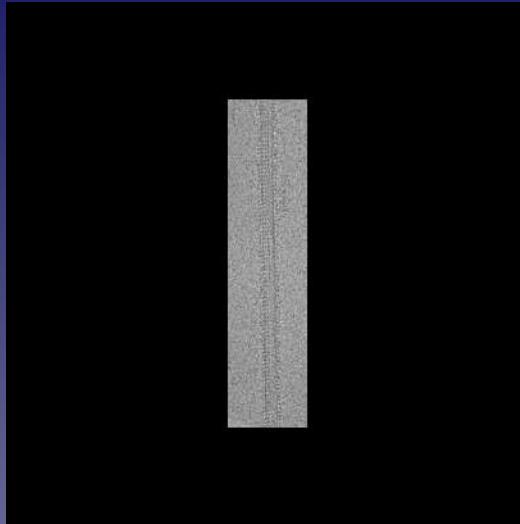


FIG. 1. The scheme presented shows the flow of data in the process of three-dimensional reconstruction.

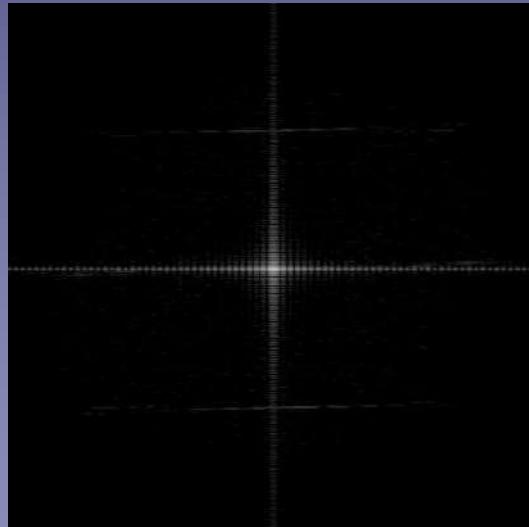
FIG. 1. Schematic diagram of the PHOELIX helical processing package. A detailed description of each step and the programs used is available as part of the PHOELIX distribution.

# Boxing & Floating Image

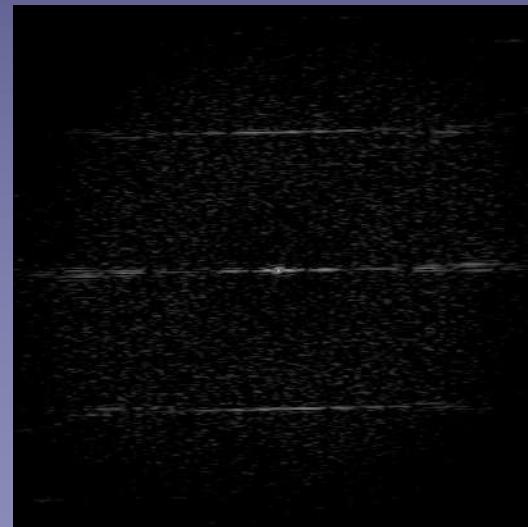
Image



FFT

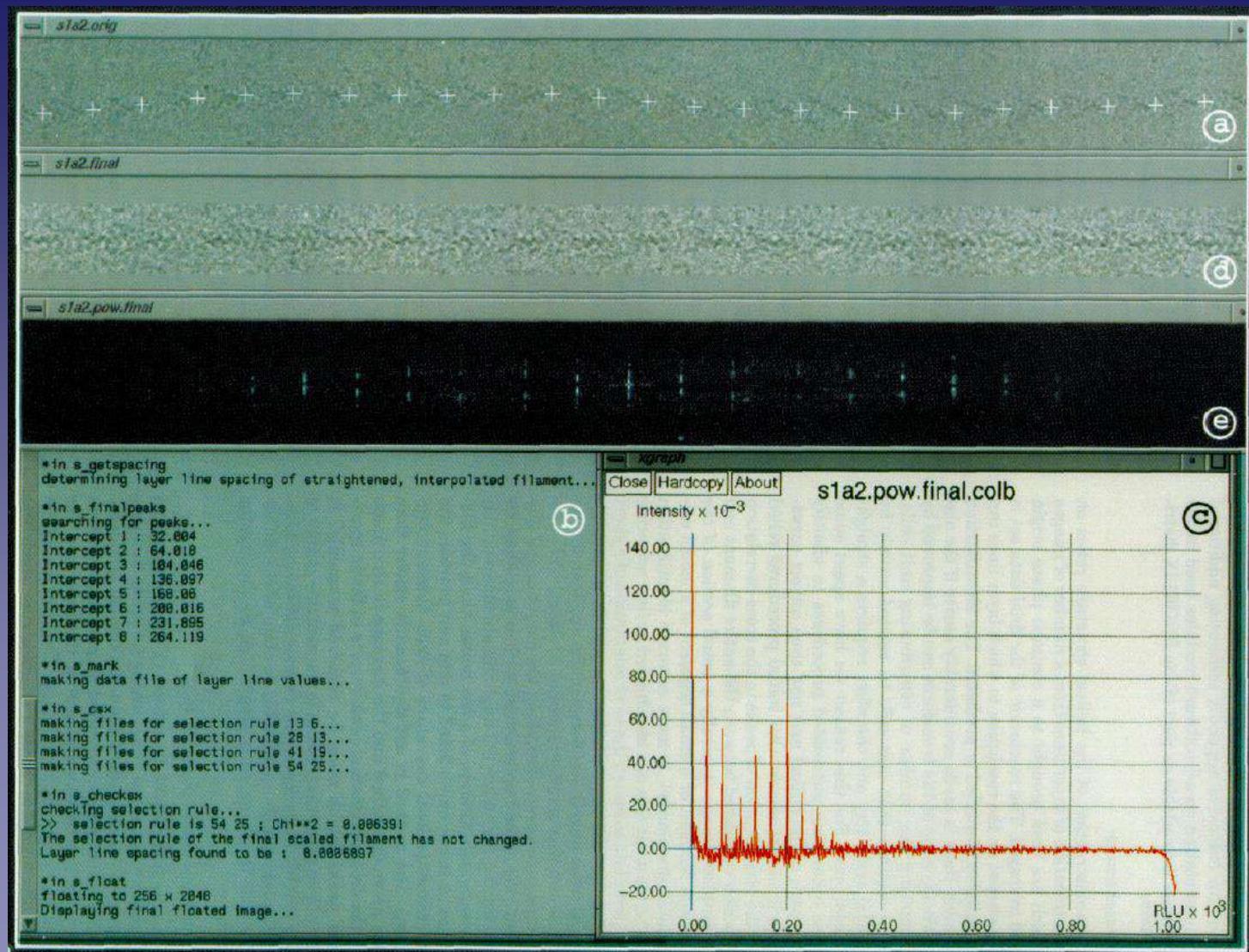


Non-Floated



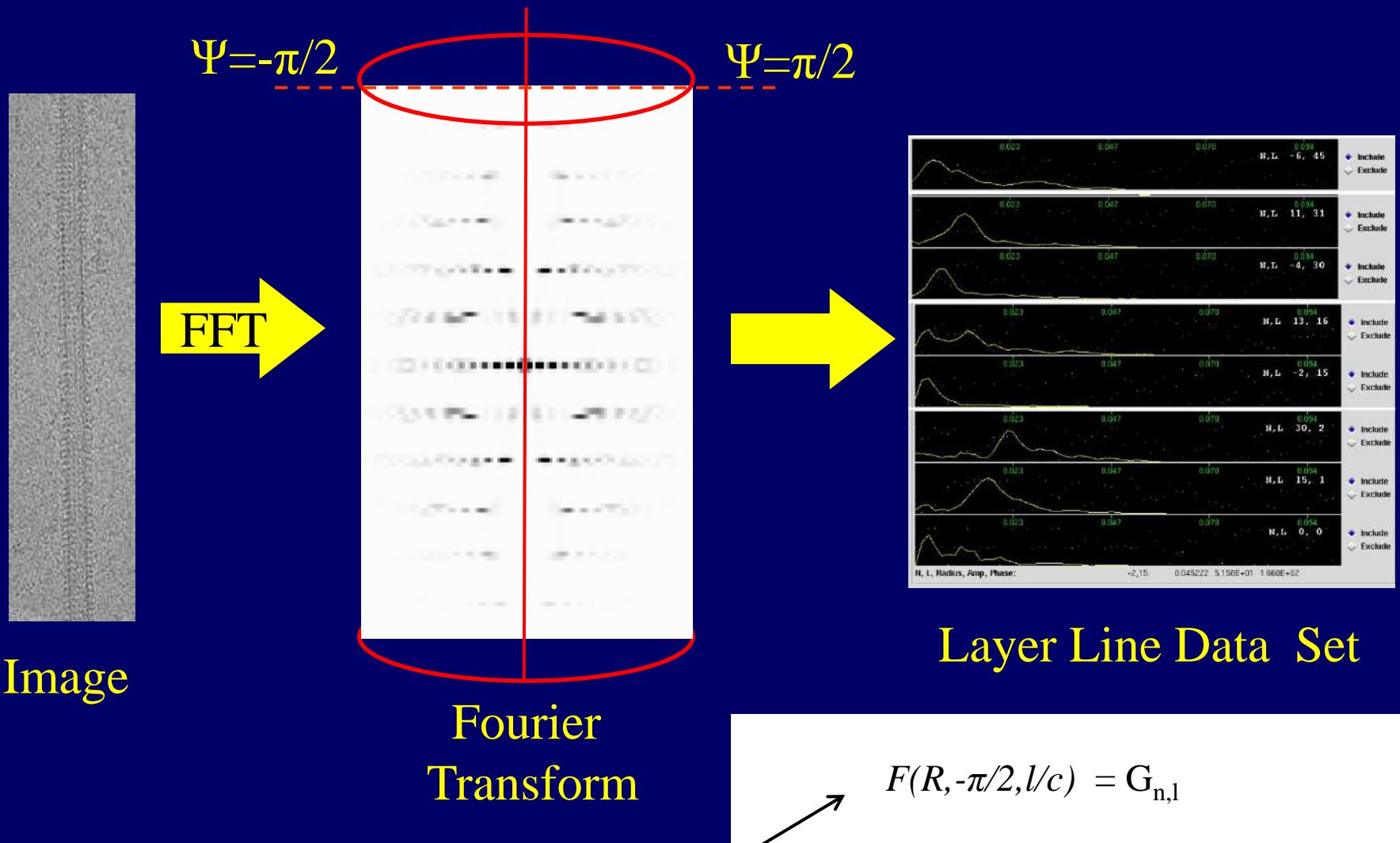
Floated

# Straightening



From: Carragher et al., JSB 116: 107-112 (1996)

# Gathering Amplitude and Phases



$$F(R, \psi, l/c) = \sum_n G_{n,l}(R) \exp [in(\psi + \frac{1}{2}\pi)], \quad \rightarrow \quad F(R, -\pi/2, l/c) = G_{n,l} \quad \text{(for even } n\text{)}$$

$$\quad \rightarrow \quad F(R, +\pi/2, l/c) = \begin{cases} G_{n,l} & \text{(for even } n\text{)} \\ -G_{n,l} & \text{(for odd } n\text{)} \end{cases}$$



Test



Printer



Firefox



UCSF-  
Chimera



### Image 1



KODAK



1



10



10



10



10

hbz\_run2.py H.S. v Nov 13 2009

Help

TIF to mrc/suprim file format

Define pixel size etc

----- Single Filament procedures -----

Power spectrum average (CTF ring inspection)

Estimate CTF parameters

Do CTF Phase flipping correction

Eliminate density gradients

Straightening filament

Normalize, apodize & pad image

Find selection rule & LL positions

Cut to integral number of repeats and reinterpolate

Extract layer lines

Create lline ranges file

Fix xshift & out of plane tilt

Make avlist file for averaging

----- Several Filament procedures -----

Edit list with files to average

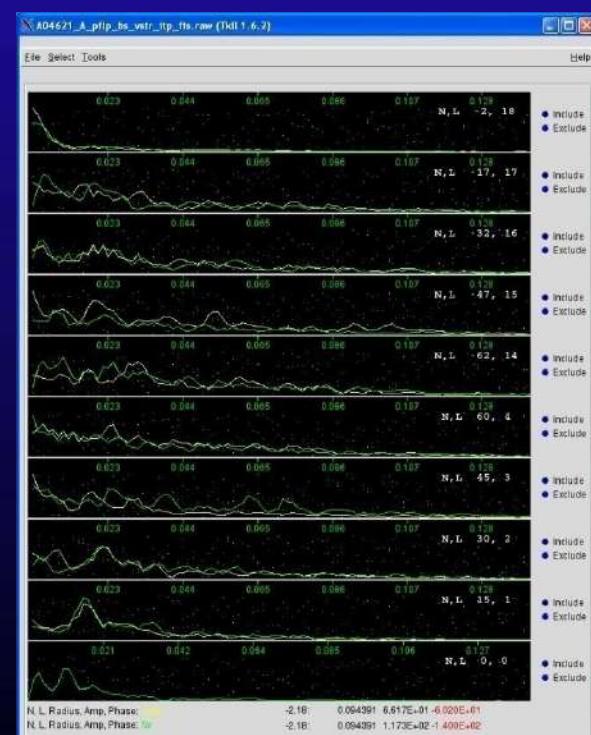
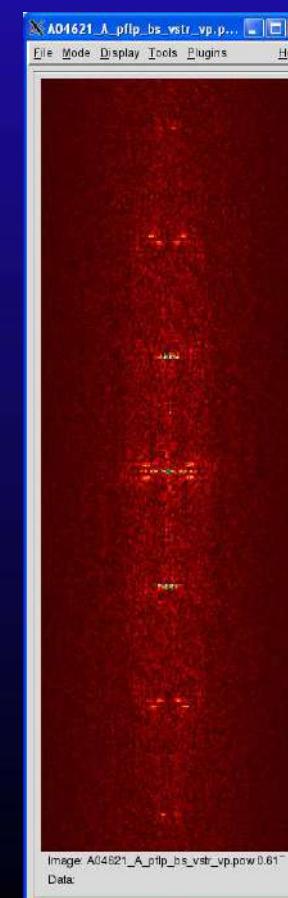
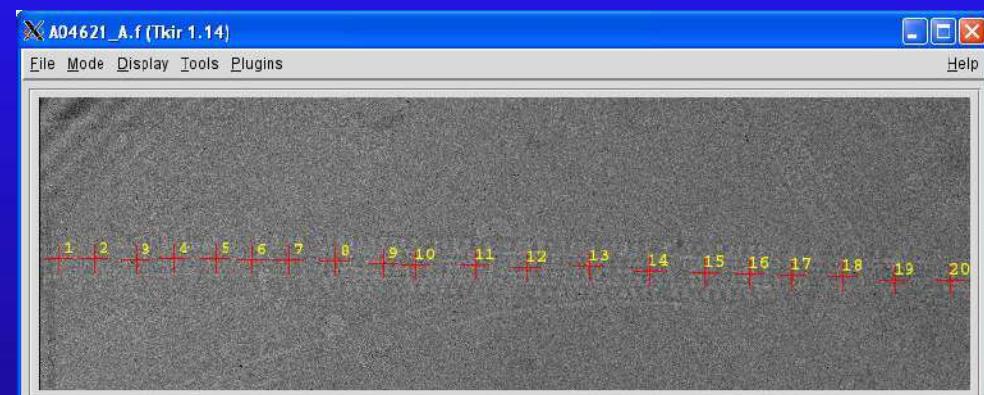
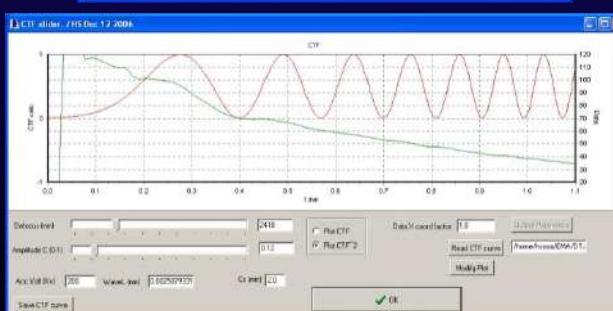
Shift ll phase origin to a template & make average

OverPlot make overplot of selected LL

----- General procedures -----

Calculate 3D map

Calculate Fourier Correlation Shell

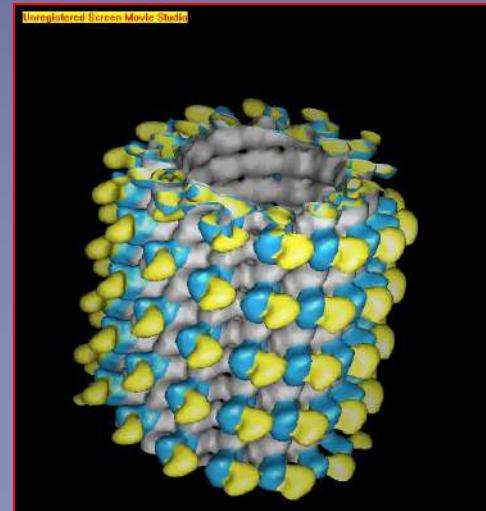


# 3D Density Map Reconstruction (Fourier Bessel inversion)

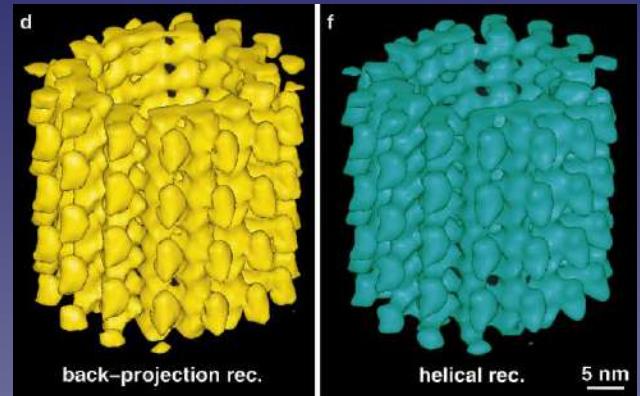
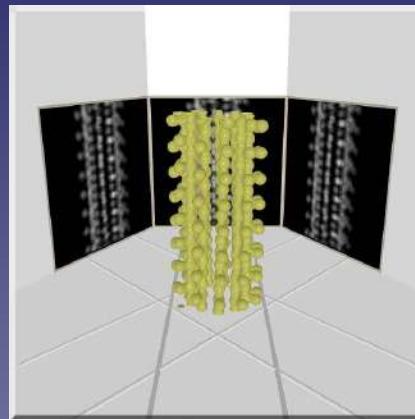
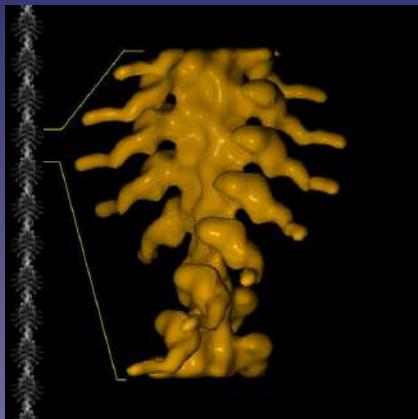
Average  $G_{n,l}$  Data set.

$$g_{n,l}(r) = \int G_{n,l}(R) J_n(2\pi R r) 2\pi R dR$$

$$\rho(r, \phi, z) = \sum_l \sum_n g_{n,l}(r) \exp(in\phi) \exp(-2\pi ilz/c)$$



# 3D Helical Reconstruction Using Real Space Methods

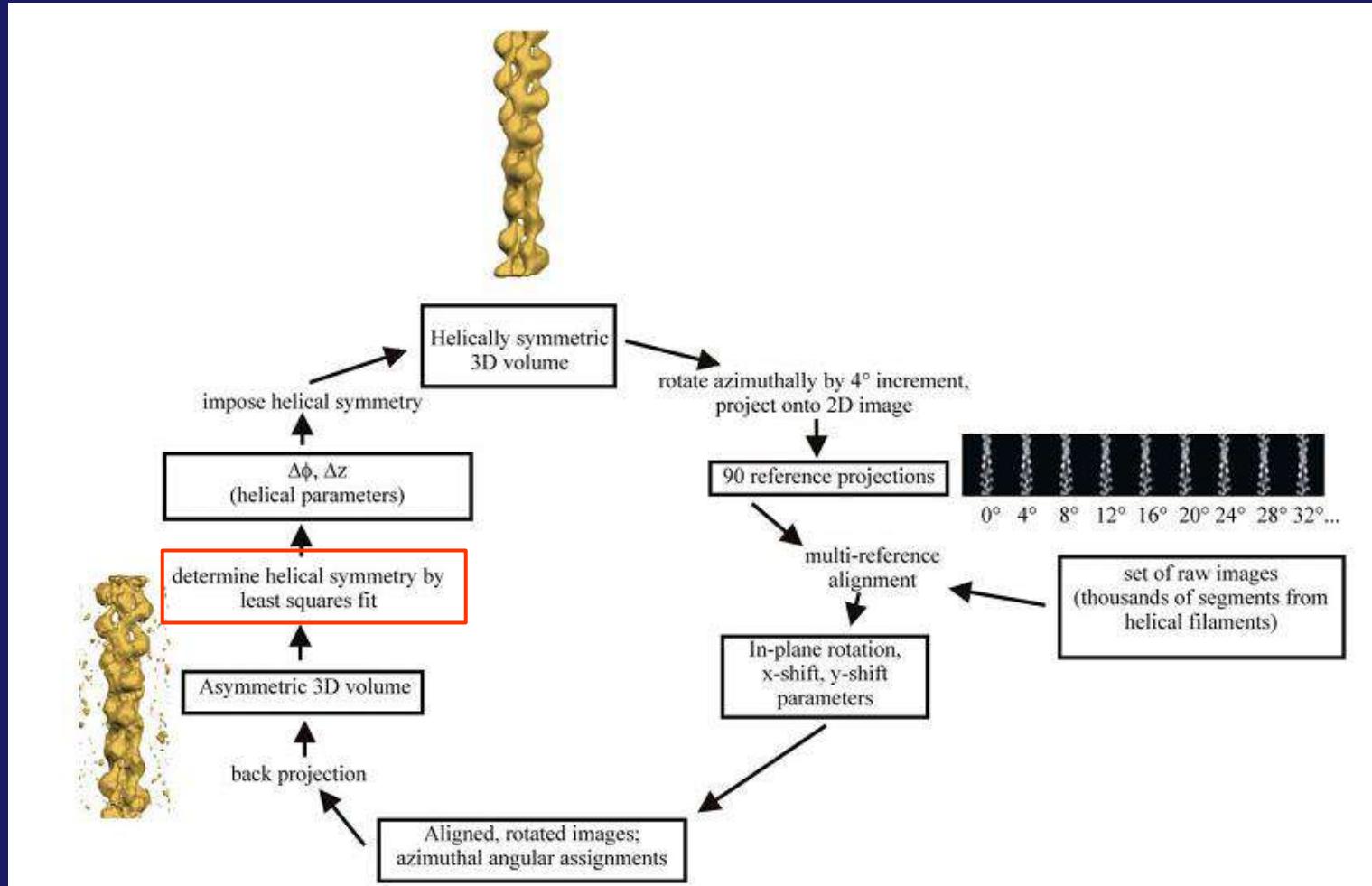


Individual images are boxed out of the filament at each asymmetric unit axial spacing and a view angle is assigned according to the helical symmetry of the filament.

A 3D volume is obtained by back-projection of the boxed images.

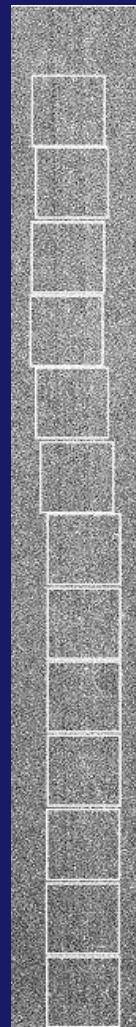
From Sosa et al. JSB 118: 149-158(1997)

# The Iterative Helical Real Space Reconstruction Method (IHRSR)

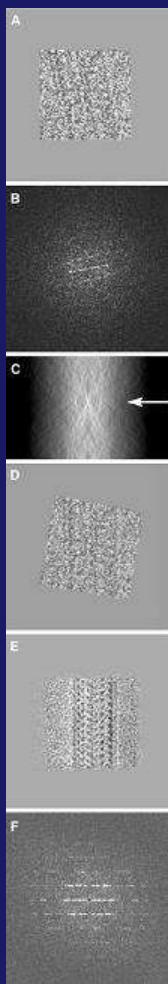


From: Egelman E.H. (2000) Ultramicroscopy 85: 225-234.

# 'Single Particle' Helical Reconstruction Methods



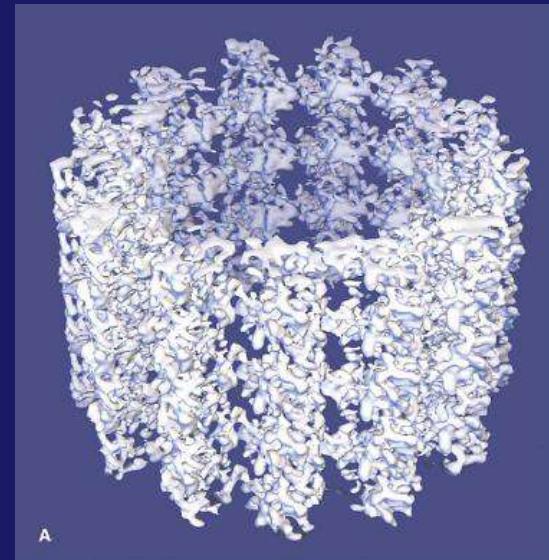
Box Filament  
segments



In-plane  
Rotational  
Alignment



Multi-reference  
alignment with atomic  
models

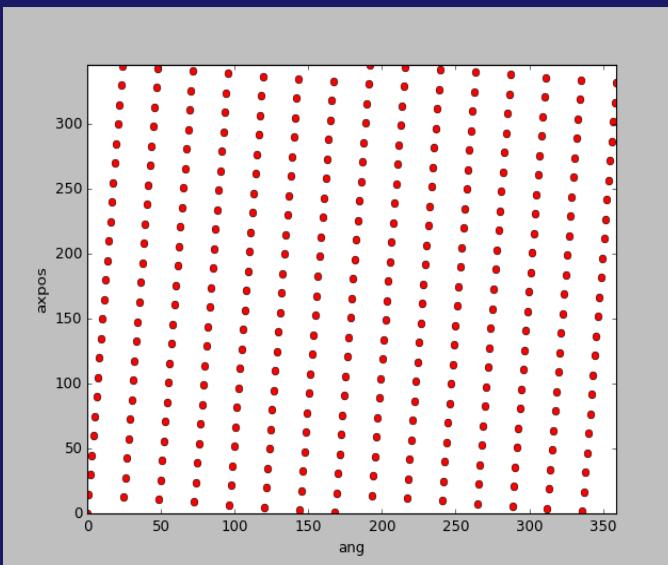


8 Å resolution  
microtubule map

From: Li et al., (2002) Structure 10: 1317-1328.

A first requirement for 3D reconstruction of a helical specimen, regardless of the method to be used (Fourier/Bessel or real space/single particle) is a good estimate of the helical parameters of the specimen:

- Rise per repeating unit ( $h$ ).
- Azimuthal rotation per repeating unit ( $\Phi$ ).



<b>l</b>	<b>n</b>
0	0
1	15
2	30
22	-17
23	-2
24	13
25	28
45	-19
46	-4
47	11
48	26
68	-21
69	-6
70	9
71	24
91	-23
92	-8
93	7
94	22

$$l = \textcolor{red}{t}n + \textcolor{blue}{u}m$$

$l$ : Layer line Number.

$t$ : Num. of turns/rep.

$n$ : Num of Helical starts & bessel order.

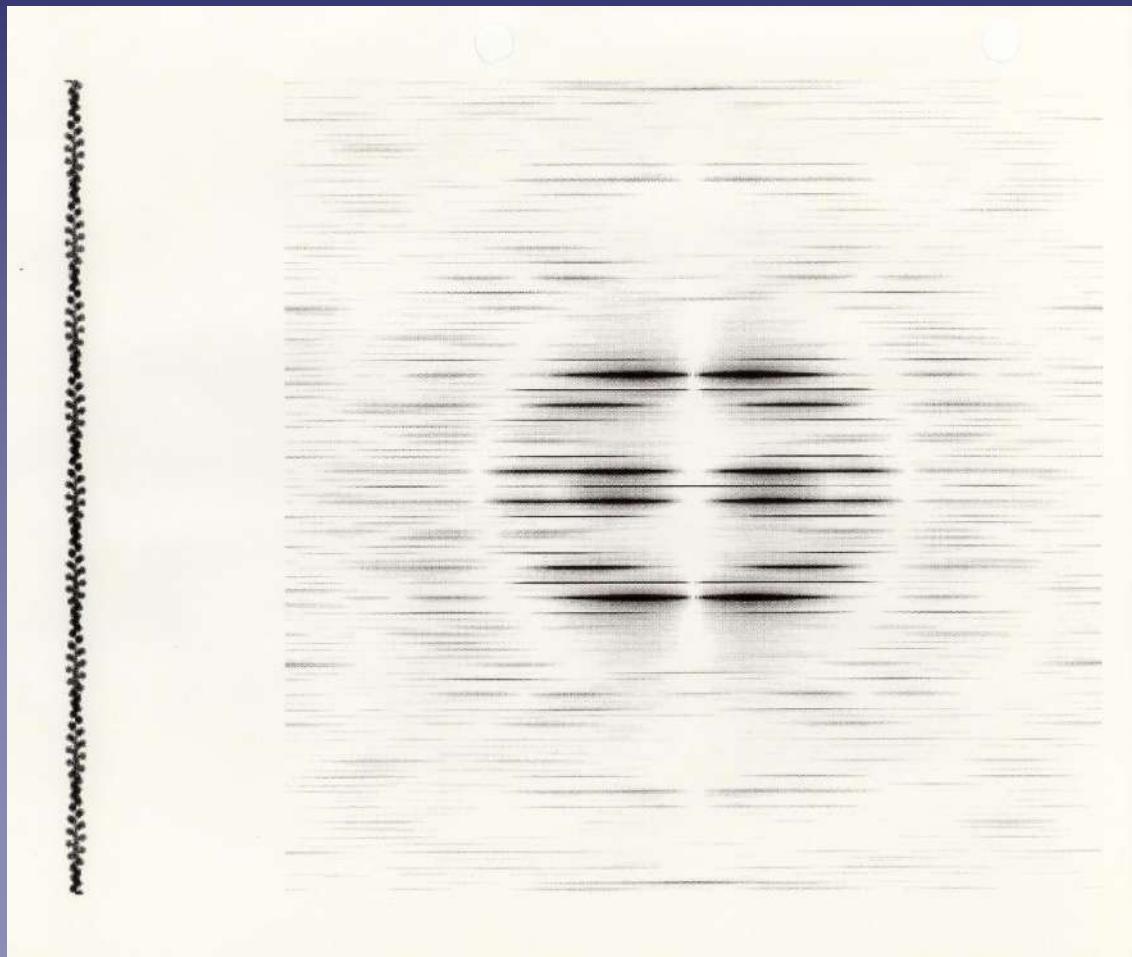
$u$ : Num. of subunits/rep

$m$ : Integer

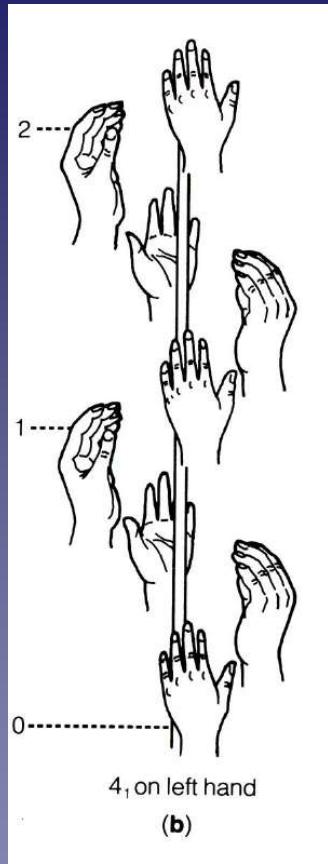
SR:  $l = 162 * n + 347 * m$   
 $\Phi :$  168.07

# Finding Helical Symmetry Selection Rule

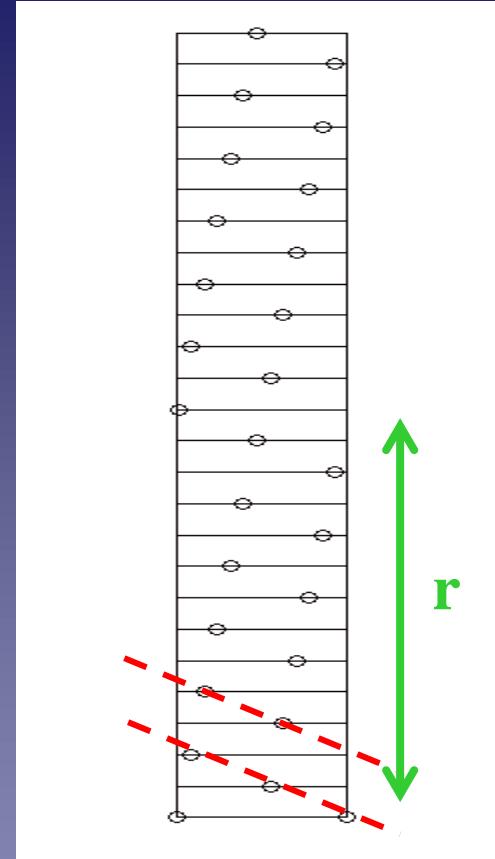
## Indexing the diffraction Pattern



# The Selection Rule



4 units in 1 Turn (RH)



13 units in 6 Turns (LH)

Knupp C, Squire JM, **HELIX: A helical diffraction simulation program**, J Appl Cryst, 2004, Vol: 37, Pages: 832 - 835

# The Selection Rule

$$l = tn + um$$

***l***: Layer line Number.  
***t***: Num. of turns/rep.  
***n***: Num of Helical starts & bessel order.  
***u***: Num. of subunits/rep  
***m***: Integer

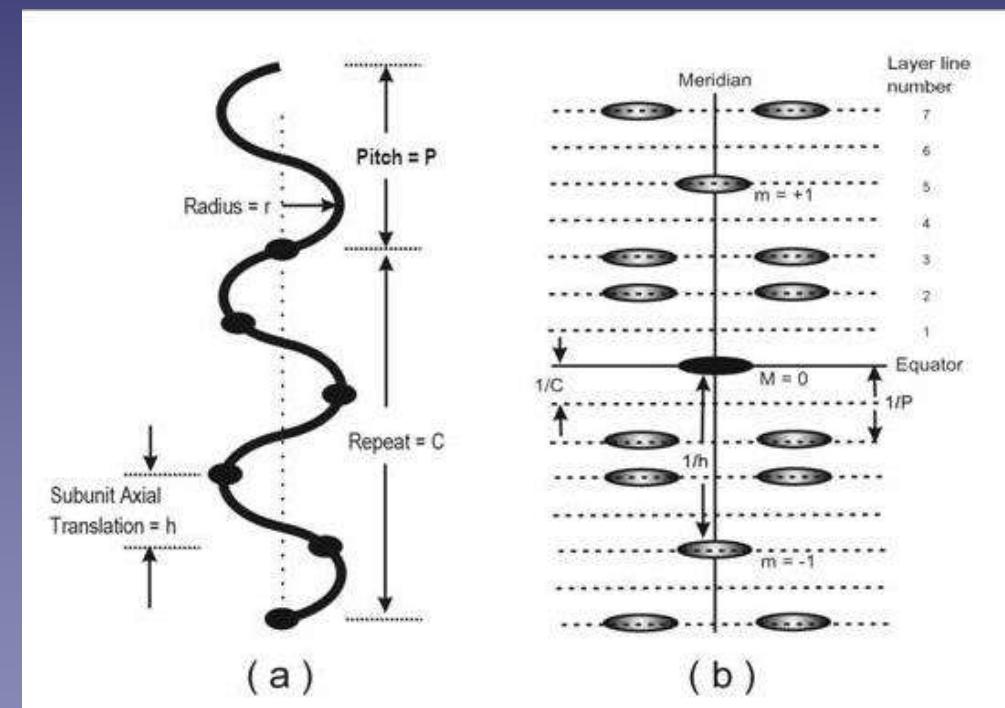
If  $k$ -fold rotational symmetry then:  
***n*** must be multiple of  $k$

$$Z = n(\phi/360^\circ)/h + m/h$$

Z: LL reciprocal spacing

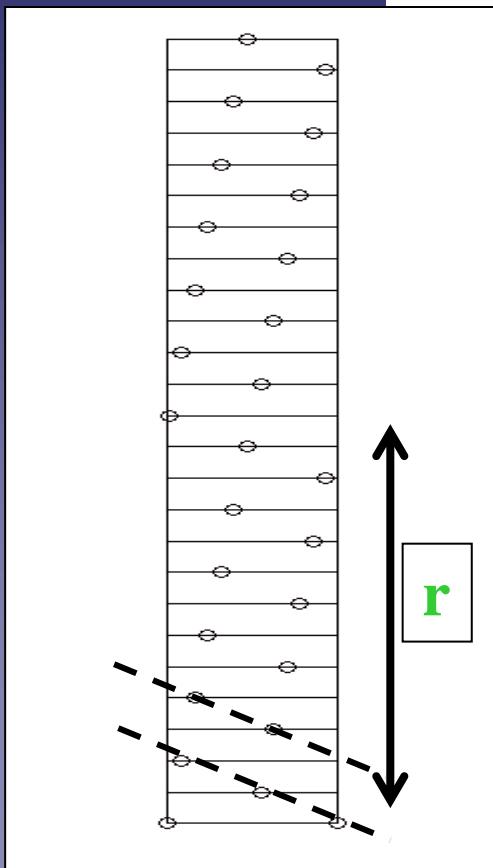
$\phi$ : twist angle.

$h$ : rise distance.



# Selection Rule Example

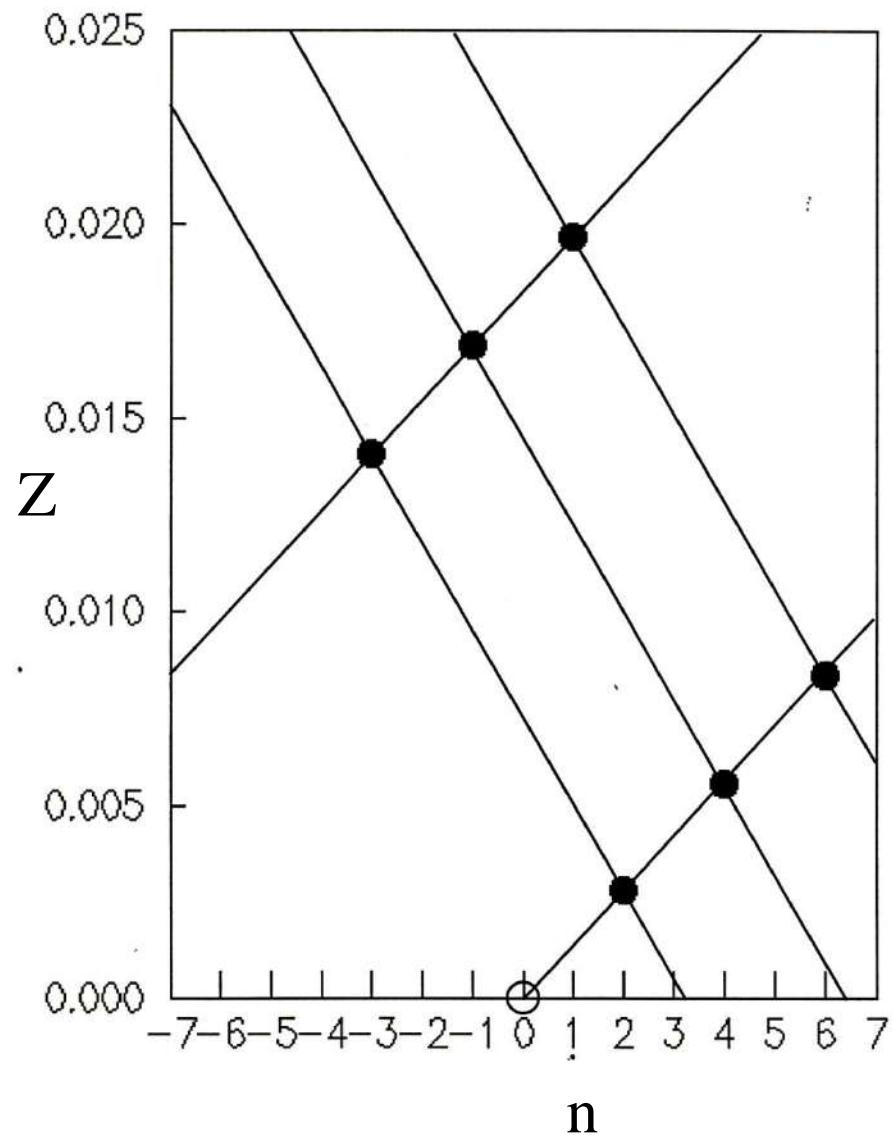
$$l = \textcolor{red}{tn} + \textcolor{blue}{um}$$



<u><math>l</math></u>	<u><math>n_{(/n) &lt;= 10}</math></u>
0	0
1	2
2	4, -9
3	6
4	-5, 8
5	-3, 10
6	-1
7	1
8	3, -10
9	5, -8
10	-6, 7
11	-4, 9
12	-2
13	0

$$l = \textcolor{red}{-6n} + \textcolor{blue}{13m}$$

**n,Z Plot for Actin**



# Software for Helical 3D Reconstruction

## Fourier-Bessel

- MRC Package
- Brandeis
- Phoelix & Suprim
- Unwin's routines
- Toyoshima's routines
- Ruby-Helix (Kikkawa's lab)
- EMIP (Stoke's lab)

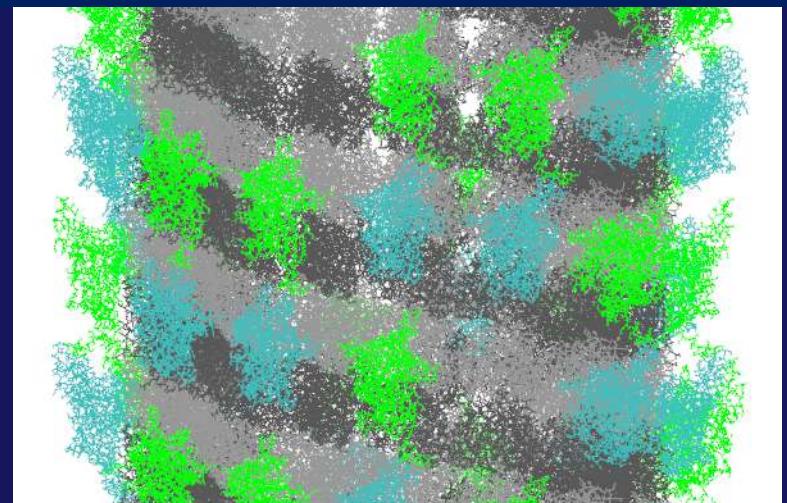
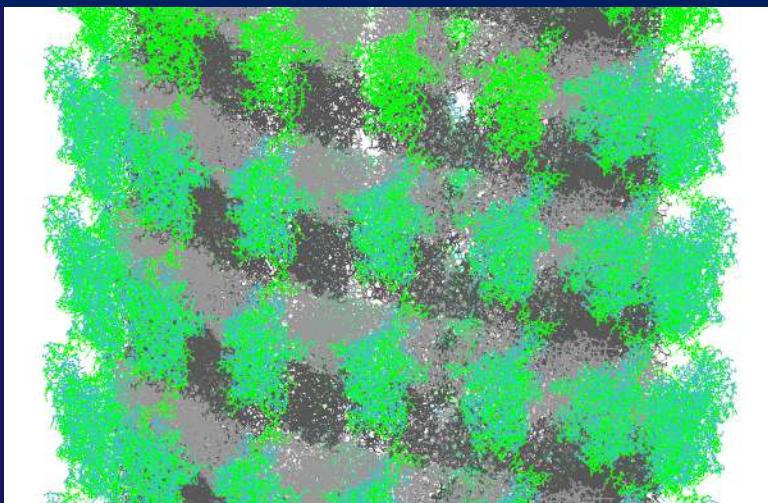
Real space or single-particle-like iterative refinement software

- FREALIGN (Grigorieff's lab)
- IHRSR (Egelman's lab)
- RELION (Schere's lab)
- SPRING (Sachse's lab)
- CryoSPARC (Structure Biotechnology Inc.)
- EMGlue (Sosa's lab)
- Spider
- IHRSR
- Frealign
- Relion
- EMAN

# 3D Helical Reconstruction Workflow

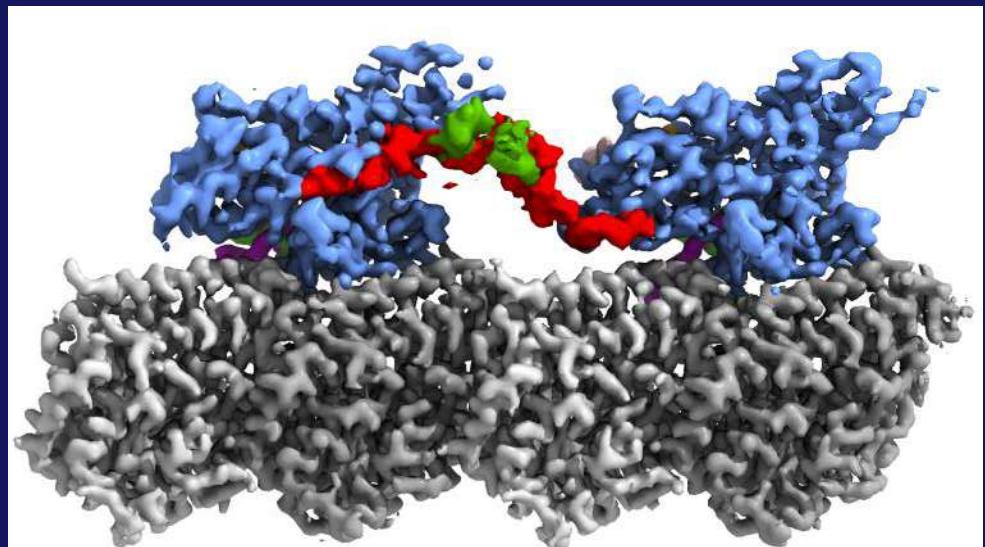
- Pick filaments & mark axis. (EMAN Boxer, Others)
  - Preliminary low resolution model (Custom)
  - Classify filaments. (Custom, SPIDER, EMAN)
  - Determine helical symmetry ( $\phi$ ,  $h$ ) (Custom)
- Extract “single particle” boxes (Spider)
- Determine 3D orientation of single particle boxes (projection matching) & make 3D reconstruction (Spider, Custom)
- Refine 3D map (IHRSR).
- Refine 3D map (FREALIGN)
- Refine 3D map (RELION)
- Model Building (UCSF-Chimera, Coot, Phenix, Direx, Modeller)

# Resolving asymmetric features in helical assemblies



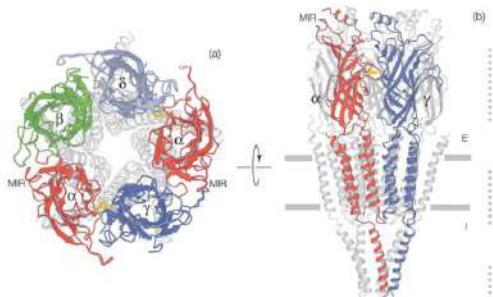
# Helical Assembly Subunit Refinement and Classification (HASRC)

1. Helical reconstruction
2. Symmetry expansion , Signal subtraction , Subunit isolation.
3. Subunit local refinement.
4. Focus 3D classification.  
(Relion 3.0)

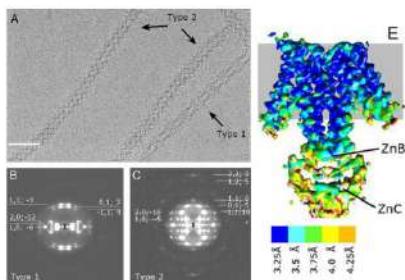


KIF14 Dimer MT complex (2.9 Å resol)  
Benoit et al., *Nat. Comm.* 12:3637 (2021)

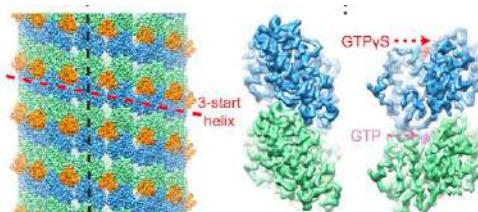
# Examples of Helical Structures @ $\leq 4 \text{ \AA}$



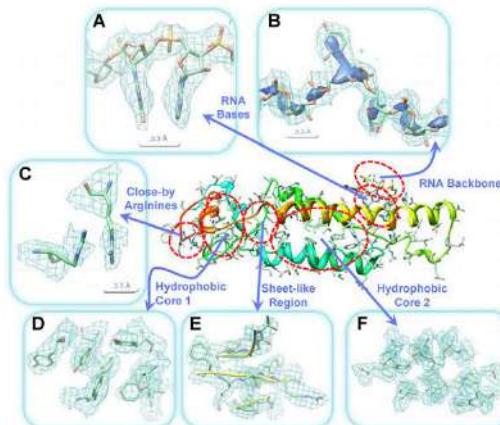
Ach Receptor ( $4 \text{ \AA}$ )  
Unwin N.(2005) JMB 346:976  
(Fourier-Bessel Method)



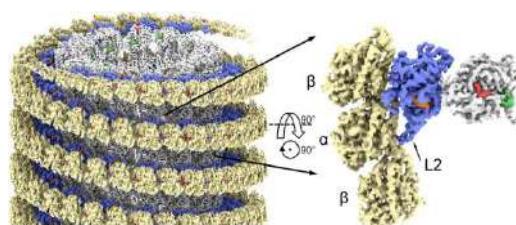
YiIP Zinc Transporter ( $4.1 \text{ \AA}$ )  
Lopez-Redondo et al., (2018)  
PNAS 115: 3042



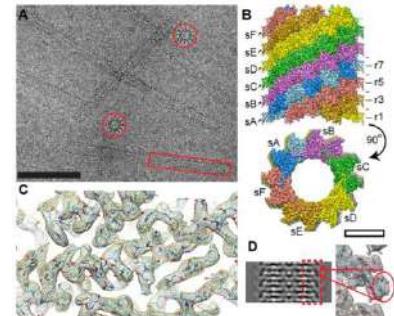
Microtubules ( $3.3 \text{ \AA}$ )  
Zhang et al., (2015)  
Cell 162: 849



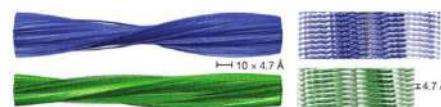
TMV ( $3.3 \text{ \AA}$ )  
Ge & Zhou. (2011)  
PNAS 108: 9637.



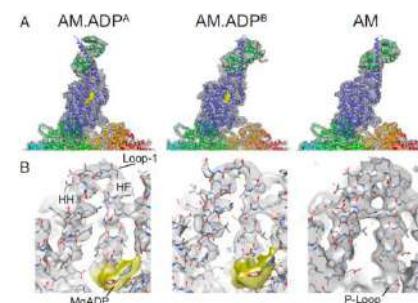
Kinesin-Microtubule complexes ( $3.5 \text{ \AA}$ )  
Benoit et al., (2018) Nature Comm.



Bacteria type VI secretion system ( $3.5 \text{ \AA}$ )  
Kudriashov et al., (2015)  
Cell 26: 952



Tau Filaments ( $3.4 \text{ \AA}$ )  
Fitzpatrick et al., (2017).  
Nature 547: 185



Acto-myosin ( $3.2 \text{ \AA}$ )  
Mentes et al., (2018). PNAS  
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# References & additional reading

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